SCHRIFTELIJK TENTAMEN

ASTROPHYSICAL HYDRODYNAMICS

3^{rd} quarter 2016/2017

April 7, 2017

NOTE: THIS EXAM CONTAINS

- 8 short questions (1-8), page 1 and 2
- 3 major questions (9-11), page 3 to 7

Grading:

- 8 short questions: each 0.5 point
- 3 major questions: each 1.5 point

Total grade is 0.5 + questions grades.

Please assure you have read all pages and questions.
Mention your name and studentnr. on ALL pages that you hand in.

Question 1.: Fluid definition

What is a fluid. Compare the relevant length scales. Describe and explain why the fluid approximation is good for the solar interior, but will not be a good description for the solar wind.

Question 2.: Lagrangian and Eulerian

What is the Eulerian description of fluid dynamics, and what the Lagrangian description. Descibe the physical meaning and difference between Eulerian derivative of a quantity Q,

$$\frac{\partial Q}{\partial t} \tag{1}$$

and

$$\frac{\mathrm{D}Q}{\mathrm{D}t} \tag{2}$$

and infer the relationship between these two derivatives for a fluid with flow field $\vec{v}$.

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Question 3.: Flow visualization
Discuss the various ways in which one can visualize fluid flow, ie. streamlines, pathlines and streaklines. Describe and sketch these various means of visualizing the flow of fluids, and explain the relationship between these.

Question 4.: Stokes' Flow theorem
Explain how the flow field in a fluid can be decomposed into three components, divergence, shear and vorticity. Give the definitions of these quantities in terms of the gradient of the flow field $\partial v_i/partialx_j$. Explain how these different components of the flow field relate to the deformation of a fluid element (use sketches/diagrams/illustrations).

Question 5.: Sound velocities
Discuss and explain the difference in phase velocity and group velocity of a soundwave. Give the definitions of these velocities, in terms of frequency ω and wave number k, and use diagrams to illustrate the different significance of these sound velocities.

Question 6.: Archimedes' principle
What does Archimedes principle state ? From the Euler equation (and Gauss law), derive Archimedes' buoyancy law. Explain how Archimedes could use this to demonstrate that a crown of gold was not made of the pure material it had been supposed to be.

Question 7.: Diffusion equation
Give the diffusion equation for a diffusion quantity Q, explain all quantities, and relate the diffusion coefficient $\mathcal{D}$ to intrinsic quantities of a fluid/gas. How does a diffusion process evolve in time (you may use sketches/diagrams). What, in effect, is viscosity in the context of a diffusion process ?

Question 8.: Supernova Remnants
The evolution of supernova remnants consists of four phases. Which are these four phases, and what is the time evolution $R(t)$ of the supernova remnant shell radius for the first two phases. Explain and discuss the physics of the different stages, and the transitions between these stages.

$$\frac{1}{\rho}\vec{\nabla}\rho = -\vec{\nabla}\cdot\vec{u}$$

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Question 9.: the Boltzmann equation and Euler equation

The Boltzmann equation specifies how the phase space density $f(\vec{x}, \vec{v}, t)$ in six-dimensional phase-space $(\vec{x}, \vec{v})$ changes in time.

a) Write down the expression for the Boltzmann equation and explain the various quantities/components of the Boltzmann equation.

b) Show that for a system where particles interact with elastic collisions, embedded in a gravitational potential Φ, and for a quantity χ that is conserved in an elastic collision, you can infer the integral equation (note: this expression uses the Einstein summation convention)

$$\int \left(\chi \frac{\partial f}{\partial t} + \chi v_k \frac{\partial f}{\partial x_k} - \chi \frac{\partial \Phi}{\partial x_k} \frac{\partial f}{\partial v_k} \right) d\vec{v} = 0 \,. \tag{3}$$

Include a rationale for the fact that the righthand side of the expression is zero !

c) Defining a local average quantity $\langle Q \rangle$ as

$$\langle Q \rangle = \frac{1}{n} \int Q f d\vec{v} \tag{4}$$

where $n = \int f d\vec{v}$ is the number density of particles, show that you can infer the following evolution equation for the average $\langle \chi \rangle$ for

$$\frac{\partial}{\partial t}(n\langle \chi \rangle) + \frac{\partial}{\partial x_k}(n\langle v_k \chi \rangle) + n\frac{\partial \Phi}{\partial x_k}\langle \frac{\partial \chi}{\partial v_k} \rangle = 0 \tag{5}$$

Subsequently, we are going to explore the Boltzmann equation for the i-th component of the fluid momentum, $\chi = m v_i$.

d) Show that you get the following equation for momentum conservation:

$$\frac{\partial}{\partial t}(\rho\langle v_i \rangle) + \frac{\partial}{\partial x_k}(\rho\langle v_i v_k \rangle) + \rho\frac{\partial \Phi}{\partial x_i} = 0 \,, \tag{6}$$

where $\rho = nm$ the mass density.

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e) If we decompose the velocity v_i into the bulk velocity u_i and the random velocity component w_i, show that

$$\langle v_i v_k \rangle = u_i u_k + \langle w_i w_k \rangle \tag{7}$$

f) By introducing two macroscopic quantities, *pressure* p and the *viscous stress tensor* π_{ik}, and defining them in terms of their microscopic significance,

$$p \equiv \frac{1}{3}\rho\langle|\vec{w}^2|\rangle \tag{8}$$

$$\pi_{ik} \equiv \rho\langle\frac{1}{3}|\vec{w}^2|\delta_{ik} - w_i w_k\rangle \tag{9}$$

infer the following macroscopic expression for the momentum equation:

$$\frac{\partial}{\partial t}(\rho u_i) + \frac{\partial}{\partial x_k}(\rho u_i u_k + p\delta_{ik} - \pi_{ik}) = -\rho\frac{\partial \Phi}{\partial x_i}\,. \tag{10}$$

g) Assuming that the viscosity term $\pi_{ik} = 0$, and rewriting the gravitational force in vector notation, $\vec{f} = -\vec{\nabla}\Phi$, rewrite expression (8) into the well-known vector form for the Euler equation.

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Question 10.: Vorticity and Kelvin Circulation Theorem

According to Stokes' flow theorem, the most general differential motion of a fluid element consists of four components.

a) Express the fluid velocity $\vec{u}(Q)$ at a point Q, removed by a small amount $\vec{R}$ from a reference position $\vec{P}$, in these four terms. Describe the physical meaning of the divergence, vorticity and shear term. You may want to illustrate this by means of a drawing.

b) Write the (differential) expressions for the divergence, shear and vorticity tensor in terms of the velocity field $\vec{u}$.

c) Vorticity can be expressed in terms of a vorticity tensor and in terms of a vorticity vector $\vec{\omega}$. Write the relation between these. How is this related to the rotational velocity $\vec{v}_{rot}$ (at a relative location $\vec{R}$) ?

d) What is barotropic flow ? Write the expression for the enthalpy h of barotropic flow. $P(\rho)=0$, $ds/dt=0$ $h = dP/\rho$

e) For a fluid with flow velocity field $\vec{u}$, write the Euler equation, taking into account the influence of a gravity field $\vec{g}$ and a pressure p.

f) Show, using the relation

$$\left(\vec{u}\cdot\vec{\nabla}\right)\vec{u} = \left(\vec{u}\times\vec{\nabla}\right)\times\vec{u} + \vec{\nabla}\left(\frac{1}{2}|\vec{u}|^2\right), \qquad (11)$$

that for a barotropic flow in a conservative gravitational field (i.e. $\vec{g} = \vec{\nabla}\phi$),

$$\frac{\partial\vec{\omega}}{\partial t} + \vec{\nabla}\times(\vec{\omega}\times\vec{u}) = 0\,. \qquad (12)$$

g) If we define the flow's circulation Γ around a circuit C (with surface area A) by

$$\Gamma = \oint_C \vec{u}\cdot d\vec{l} = \int_A \vec{\omega}\cdot\vec{n}\,dA, \qquad (13)$$

how does Γ change with time if

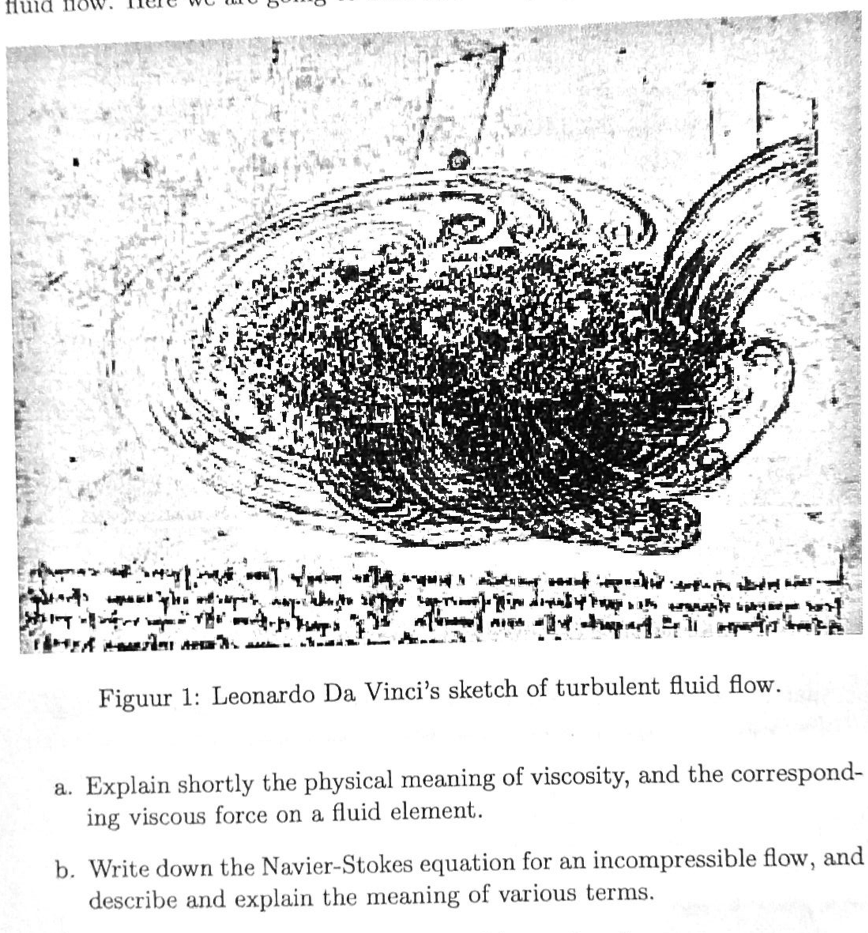
$$\frac{d\Gamma}{dt} = \int_A \frac{\partial\vec{\omega}}{\partial t}\cdot\hat{n}\,dA + \oint_C (\vec{\omega}\times\vec{u})\cdot d\vec{l}. \qquad (14)$$

The resulting relation is Kelvin's circulation theorem.

h) Explain the meaning of Kelvin's circulation theorem.

Question 11.: Turbulence

In many astrophysical environments we are confronted with turbulent fluid flow. Here we are going to look at a few properties.



Figuur 1: Leonardo Da Vinci's sketch of turbulent fluid flow.

a. Explain shortly the physical meaning of viscosity, and the corresponding viscous force on a fluid element.

b. Write down the Navier-Stokes equation for an incompressible flow, and describe and explain the meaning of various terms.

c. What is the definition of the Reynolds number ?

d. List at least five characteristics of turbulent flows (there are at least seven).

e. Describe how a flow transits from a low Reynolds number flow to a high Reynolds number flow, and illustrate this by means of the outflow from a jet (also see figure 2).

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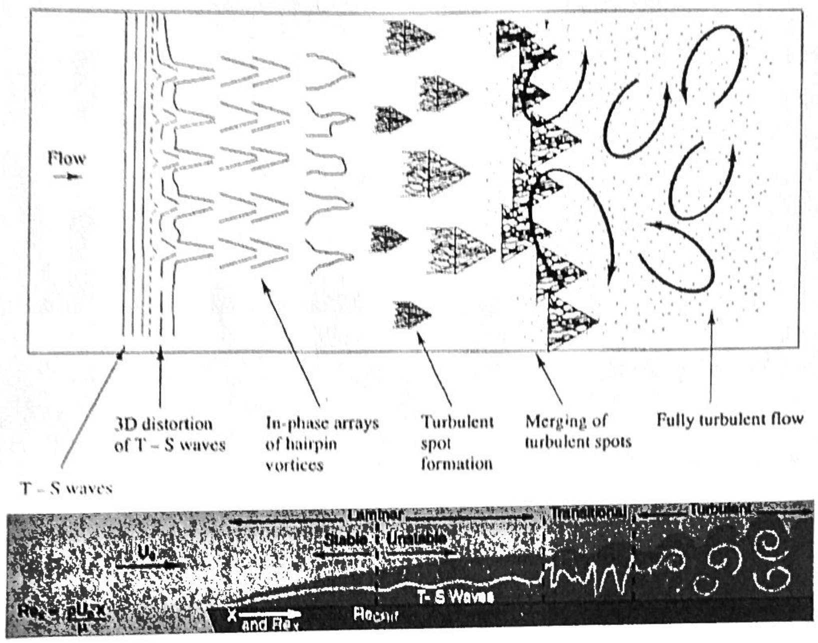

Figuur 2: Formation and Evolution of Turbulent Flow along a flat plate.

f. Turbulent flow usually involves vorticity, marked by a hierarchy of eddies. Kolmogorov's theory describes the energy distribution along the spectrum of eddies. Draw the Kolmogorov energy spectrum $E(k)$ as a function of wavenumber k, and indicate which three different regimes/length scales you can recognize.

g. Write the expression for the Kolmogorov energy spectrum $E(k)$, emphasizing the power law regime. Which scales contain most energy, and on which scales is most energy dissipated ?

$E = \left(\frac{l_0}{\lambda}\right)$

h. There are theories of star formation that ascribe it to turbulence in molecular clouds. Describe how this process would work.

SUCCES !!!!
BEDANKT VOOR JULLIE AANDACHT EN INTERESSE !!!!

Rien & Saikat